*Mathematical Problems in Climate Dynamics, CIMA + IFAECI*

Univ. of Buenos Aires,
2–13 November 2018

# *Basic Facts of GFD + Atmospheric LFV, Wind-driven Oceans, Paleoclimate & "Tipping Points"*


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*Please visit these sites for more info.*

https://dept.atmos.ucla.edu/tcd, http://www.environnement.ens.fr/
and https://www.researchgate.net/profile/Michael_Ghil

# *Overall Outline*

- **Lecture I: Observations and planetary flow theory (GFD[⌘])**
- **Lecture II: Atmospheric LFV[*] & LRF[**]**
- **Lecture III: EBMs[+], paleoclimate & "tipping points"**
- **Lecture IV: Nonlinear & stochastic models—RDS[❖]**
- **Lecture V: Advanced spectral methods–SSA[±] *et al*.**
- **Lecture VI: The wind-driven ocean circulation**

(⌘) GFD = Geophysical fluid dynamics
(*) LFV = Low-frequency variability
(**) LRF = Long-range forecasting
(+) EBM = Energy balance model
(❖) RDS = Random dynamical system
(±) SSA = Singular-spectrum analysis

# *Lecture II: Atmospheric Low-Frequency Variability (LFV) & Long-Range Forecasting (LRF)*

## *Outline*

1. Observations of persistent anomalies
   - Blocked & zonal flows
   - Characteristics of persistent anomalies
2. The deterministic chaos paradigm
   - Forced dissipative systems
   - Succesive bifurcations
   - Predictability and prediction
3. "Waves" vs. "particles"
   - Multiple regimes & Markov chains
   - Oscillatory modes & broad spectral peaks
   - Which one is it & how does that help?

# *Lecture II: Outline*

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# "Limited-contour" analysis for atmospheric low-frequency variability

*10-day sequences of*
*subtropical jet paths:*
*blocked vs. zonal*
*flow regimes*

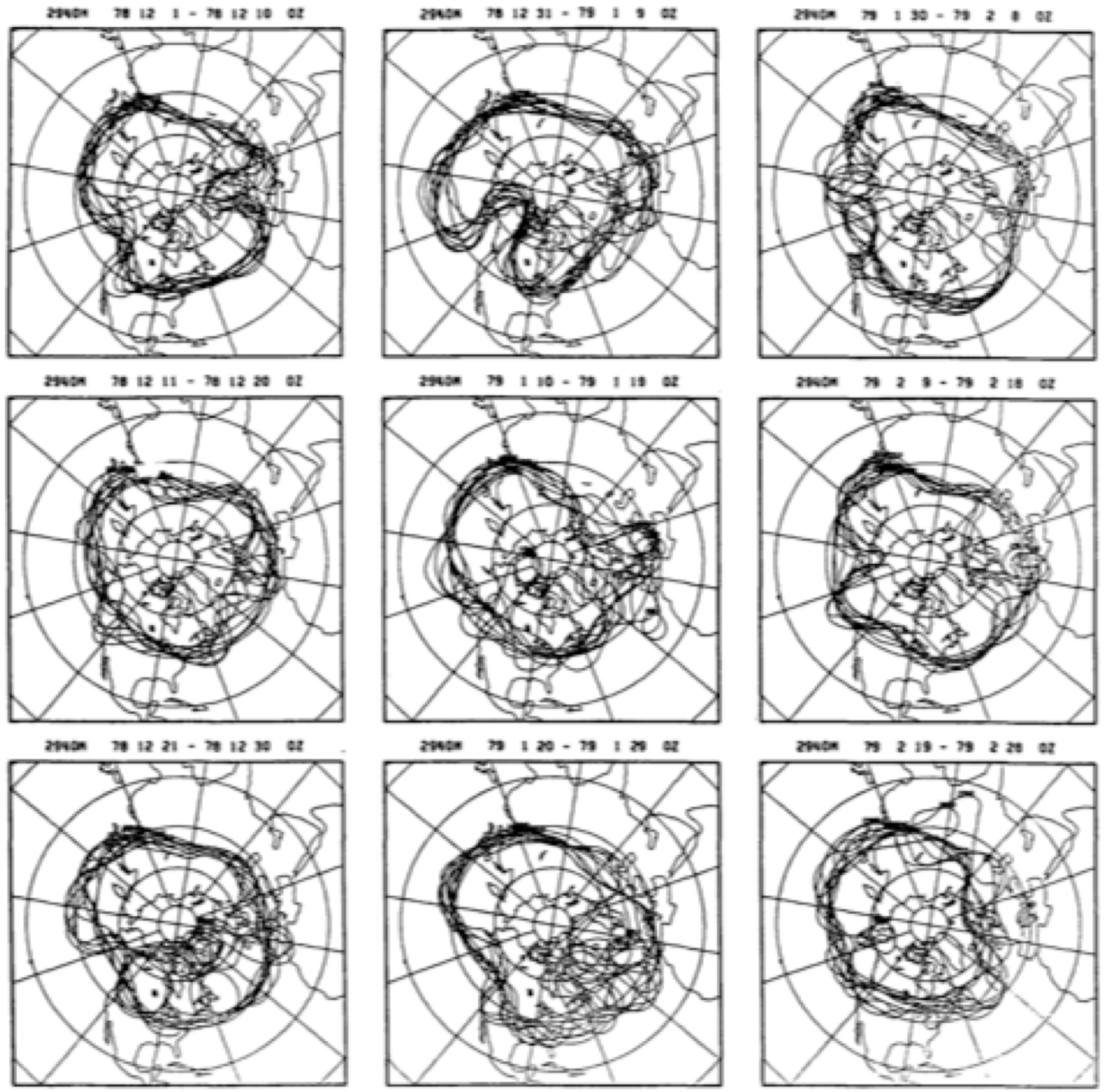

FIG. 1. Limited contour analysis of Northern Hemisphere (NH) flows. Daily contours of a prescribed height (2940 m in this case—roughly corresponding to the jet axis) are superimposed for successive 10-day intervals during NH winter 1978/79. Persistence is illustrated by some of the panels (see text for details).

**Kimoto & Ghil, *JAS*, 1993a**

# Transitions Between Blocked and Zonal Flows
## in a Barotropic Rotating Annulus with Topography

**Zonal Flow**
13–22 Dec. 1978

**Blocked Flow**
10–19 Jan. 1963

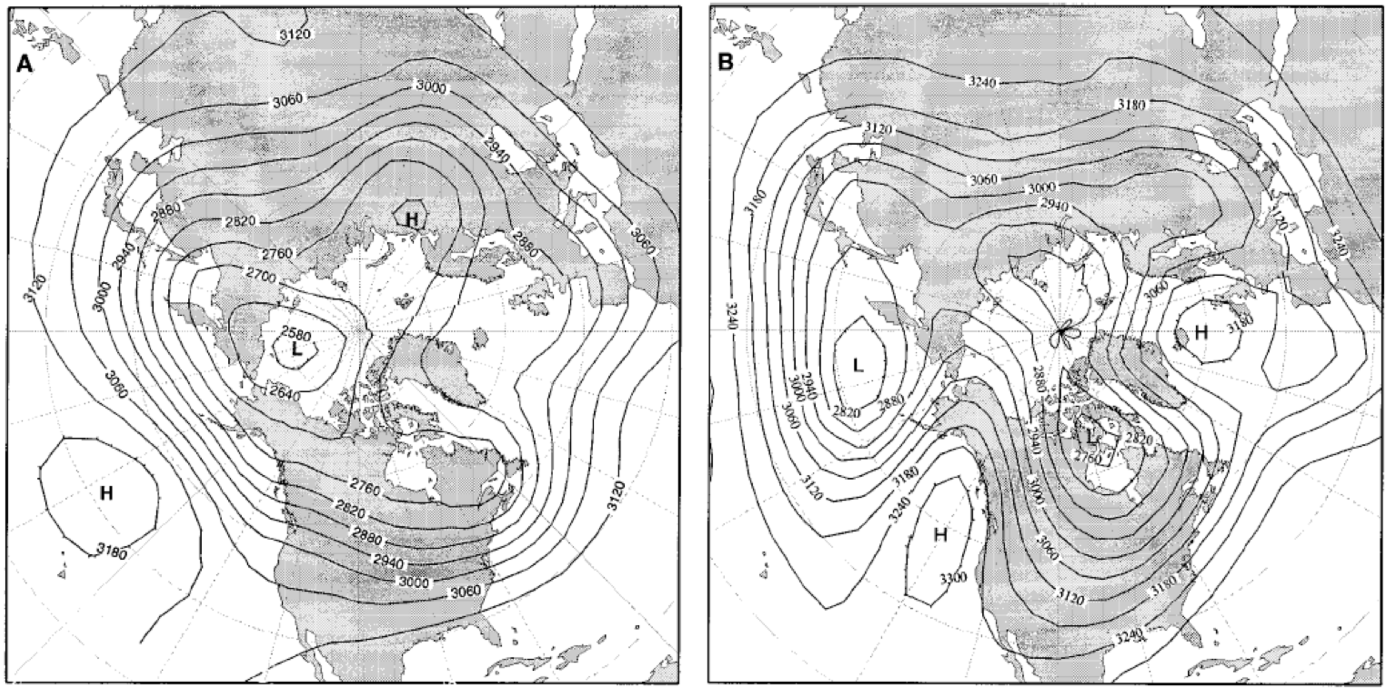


**Fig. 1.** Atmospheric pictures of (**A**) zonal and (**B**) blocked flow, showing contour plots of the height (m) of the 700-hPa (700 mbar) surface, with a contour interval of 60 m for both panels. The plots were obtained by averaging 10 days of twice-daily data for (A) 13 to 22 December 1978 and (B) 10 to 19 January 1963; the data are from the National Oceanic and Atmospheric Administration's Climate Analysis Center. The nearly zonal flow of (A) includes quasi-stationary, small-amplitude waves (32). Blocked flow advects cold Arctic air southward over eastern North America or Europe, while decreasing precipitation in the continent's western part (26).

Weeks, Tian, Urbach, Ide, Swinney, & Ghil (*Science*, 1997)

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# *Characteristics of intraseasonal variability (~ atmospheric LFV)*

1. **Geographically fixed appearance and regional character** [*]

   **("*teleconnections*" – Wallace & Gutzler, 1981)**

2. **Persistence**

   **(*persistent anomalies* – Dole, 1982, 1986; Horel, 1985)**

3. **Recurrence**

   **(*multiple regimes* – Mo & Ghil, 1987, 1988; Kimoto & Ghil, 1993a,b)**

4. **Barotropic structure**

   **( barotropic, or 3$^{rd}$, adjustment; see next page)**

[*] **but Branstator (1987) & Kushnir (1987), 25-day hemispheric wave;**
**Benzi et al., 1984 +, hemispheric bimodality;**
**Wallace, Thompson & co. – Arctic Oscillation.**

# *Barotropization*

## – barotropic (3rd) adjustment(*)

**(a) statistical theory of turbulence**

**(Charney, 1971; Rhines, 1979; Salmon, 1980)**

**(b) evolution of baroclinic eddies & "wave maker"**

**(Hoskins & Simmons, 1978; Green-Illari-Shutts)**

**(c) external Rossby wave, & its instability**

**(Held-Panetta-Pierrehumbert, 1985–87)**

**(*)After hydrostatic (1st) and baroclinic (2nd) adjustment.**

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# *Forced dissipative systems*

Most fluid dynamical problems — and many other problems in biology, chemistry, and continuum physics— lead to ODEs (or equivalent PDEs) of the form

$$\dot{x}_i = a_{ijk} x_j x_k - b_{ij} x_j + c_i \, , \quad i = 1, 2, \ldots, N \, . \qquad \text{(FD)}$$

Here we used the summation convention for repeated indices. In fluid-flow problems, the quadratic terms in (FD) above represent the nonlinear advection term $\vec{u} \cdot \nabla \vec{u}$ . This term is associated with the Jacobian in the QG equation.

The above equation is *autonomous* and it has unique solutions for all initial data (ID) $x(0) = x_0$; these solutions depend continuously on the ID, $x = x(t; x_0)$. When the solutions exist for all times, $-\infty < t < \infty$ (*), then Eqs. (FD) define a *differentiable dynamical system* (DDS). In particular, we shall assume that this system is *forced*, $c_i c_i \neq 0$, and *dissipative*, $b_{ij} x_i x_j > 0$.

N.B. The quadratic terms are necessarily *energy conserving* if $a_{ijk} = -a_{ikj}$. and the orbits of (FD) describe a flow in the phase space of $\{x_i, i = 1, \ldots, N\}$..

(*) *Counterexample*. The solutions of $\dot{x} = x^2$ are unique and depend continuously on $x_0$ but they blow up at $t = 1$!

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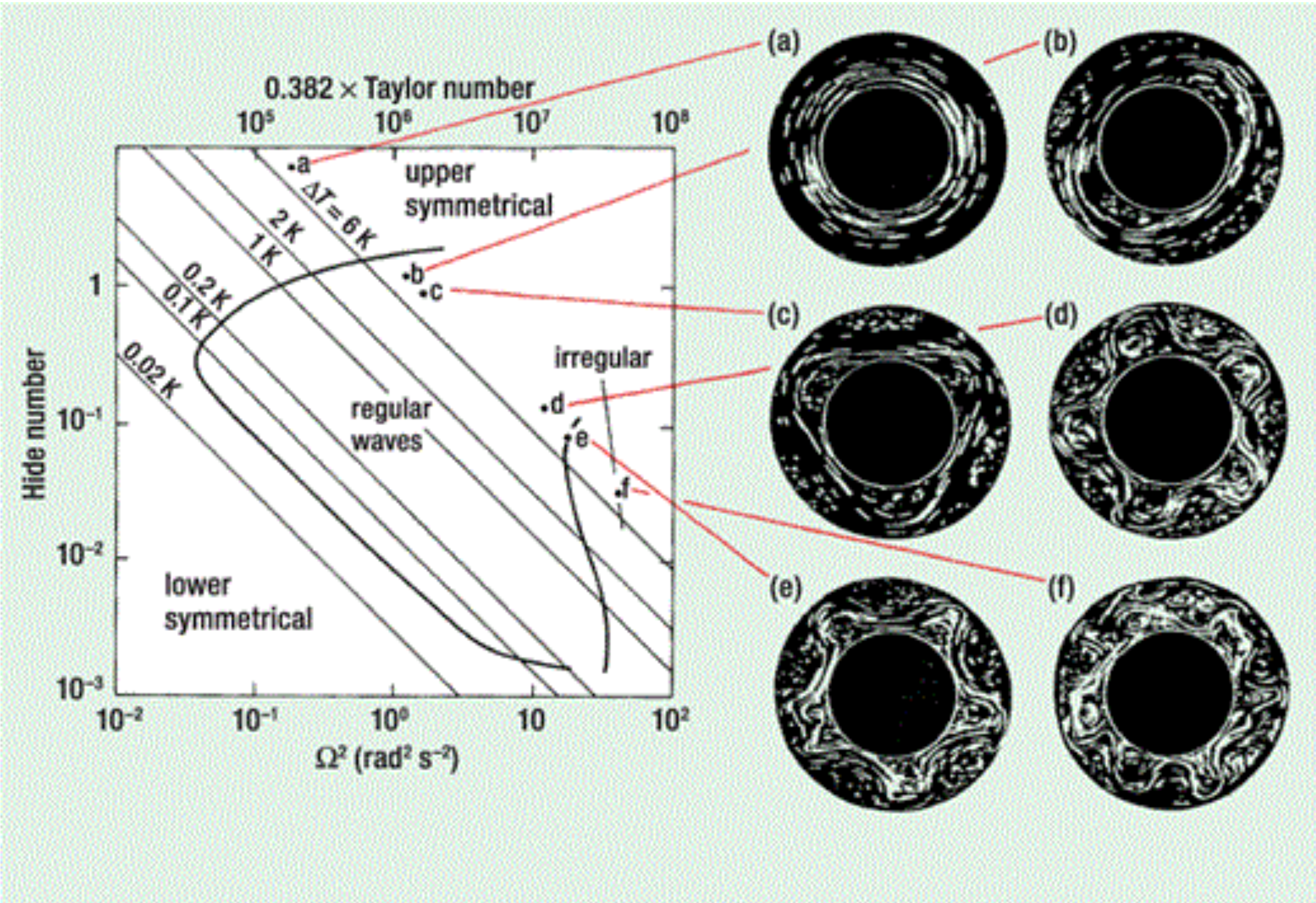
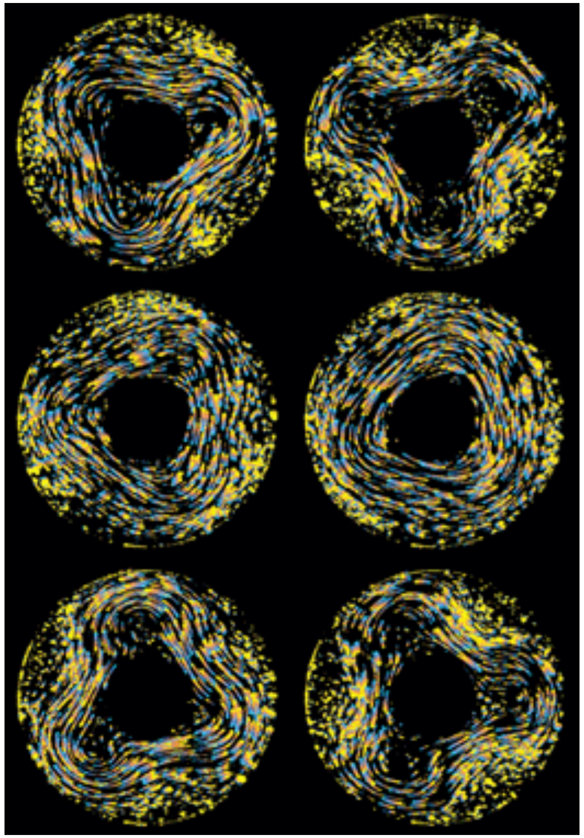
# Flow patterns and the regime diagram

Successive transitions from higher to lower symmetry of the flow pattern, in space and time, as the rotation rate Ω increases: from steady-state, axisymmetric (Hadley regime), via purely periodic in space and time (steady waves, Rossby regime) and doubly-periodic vacillation (amplitude, shape), on to irregular, quasi-geostrophic (QG) turbulence.

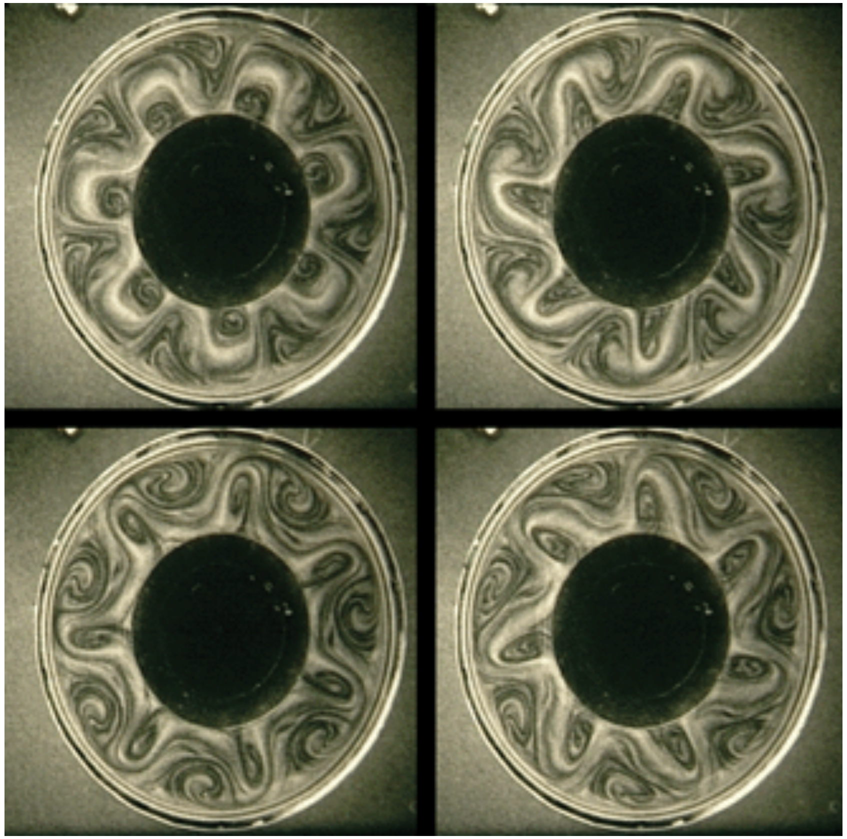


# Regime diagram – experimental

For a fixed apparatus (height $D$, gap width $L = b–a$) and fluid (expansion coefficient α, viscosity ν), one can change the rotation rate Ω and the temperature difference $\Delta T = T_b – T_a$. As Ω increases, we move along a given straight, downward-slanting diagonal, to the right and down; as $\Delta T$ increases, we move from one diagonal to another, to the right and up. The heavy contours represent sharp transitions from one regime to another one. These transitions are now associated with bifurcations.



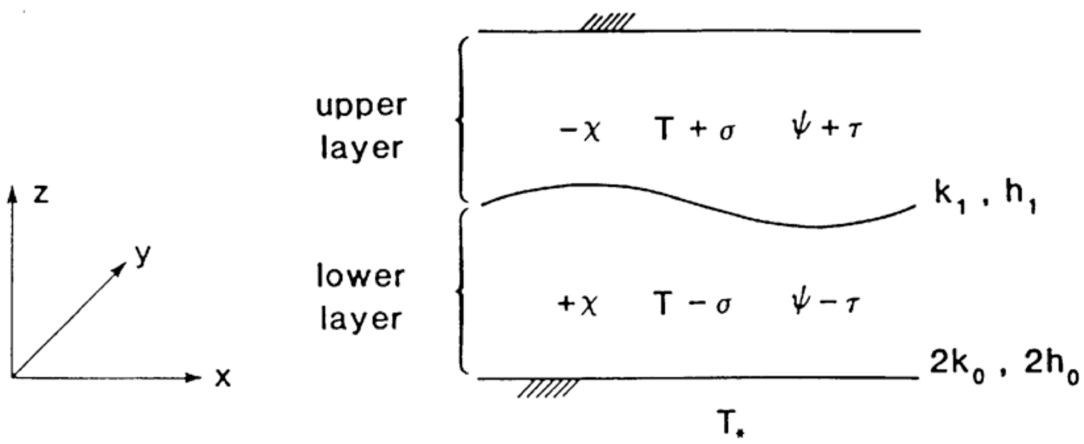
Amplitude vacillation



Tilted-trough vacillation

# Lorenz (1963b) model: Mechanics of vacillation

Lorenz was motivated by the atmospheric index cycle (Rossby, 1939; Namias, 1950), but clearly inspired by the rotating annulus results.
It is the latter that he was modeling in this paper; see also Lorenz (1967).



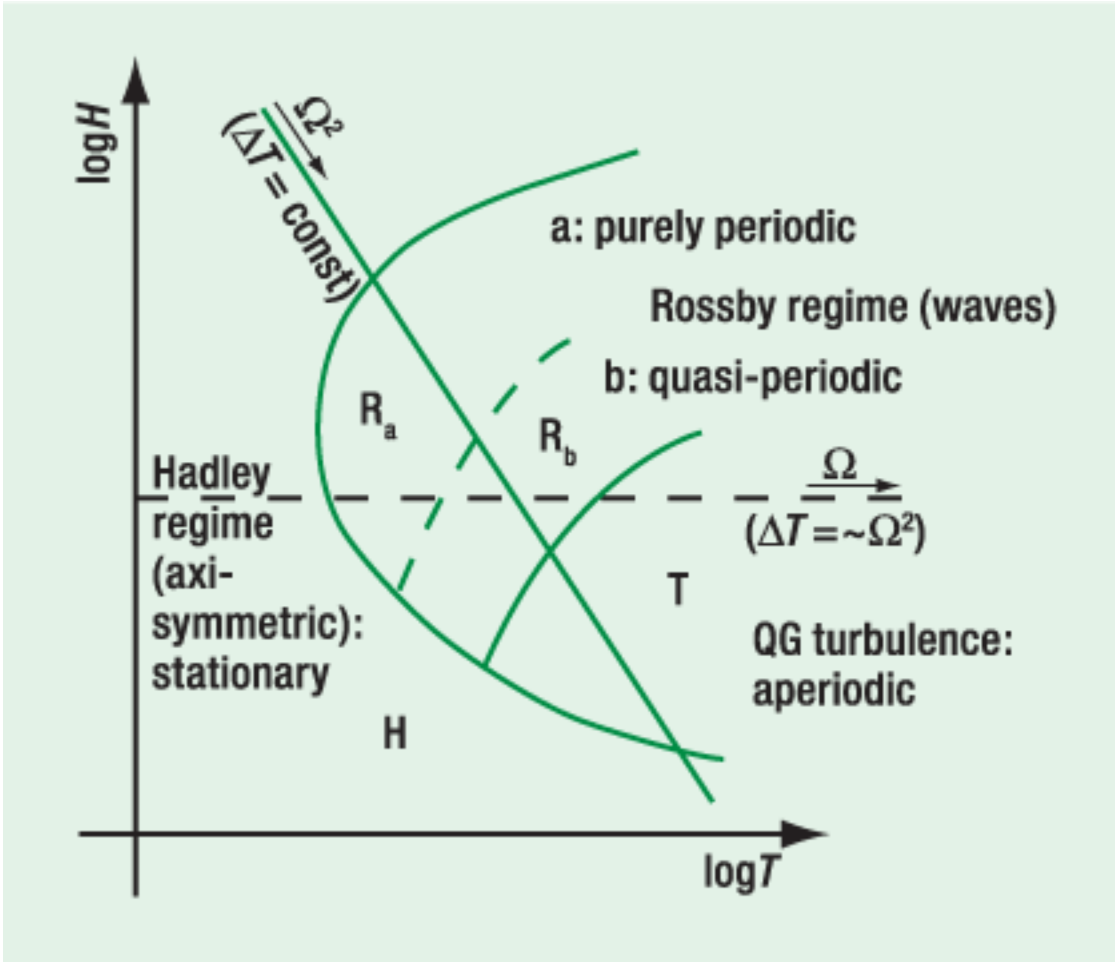

With $f =$ const., $\overline{g(x,y)}$ the horizontal mean of $g$, and $\sigma(x,y,t) = \overline{\sigma}(t)$ one has

$$
\begin{aligned}
\partial\nabla^2\psi/\partial t &= -J(\psi,\nabla^2\psi) - J(\tau,\nabla^2\tau) && , \\
\partial\nabla^2\tau/\partial t &= -J(\psi,\nabla^2\tau) - J(\tau,\nabla^2\psi) && +f\nabla^2\chi, \\
\partial T/\partial t &= -J(\psi,T) && +\overline{\sigma}\nabla^2\chi, \\
\partial\overline{\sigma}/\partial t &= && -\overline{T\nabla^2\chi}.
\end{aligned}
\qquad \text{(L63b)}
$$

Here $J(g,h) = g_x h_y - g_y h_x$ is the Jacobian and the thermal wind equation $\nabla^2 T = A\nabla^2 T$ closes the system (L63b), with $A$ depending on the fluid.

# Regime diagram – simplified

Lorenz (1963b) studied a truncated model of 14 ODEs. He essentially obtained the first few bifurcations, up to and including the quasi-periodic, vacillation regime. Beyond that, the low-order truncation prevents one from reaching the QG turbulence.
Today, such studies can be carried out on the full system of high-resolution equations.



# General idea

As we push the system harder, it responds by coming up with more complex responses, i.e., it loses symmetry in both time & space. ***In time***, it may go from being in steady state to being periodic and then chaotic; ***in space***, it often goes from being homogeneous to periodic and then to irregular. thus, the two kinds of symmetry loss are interrelated.

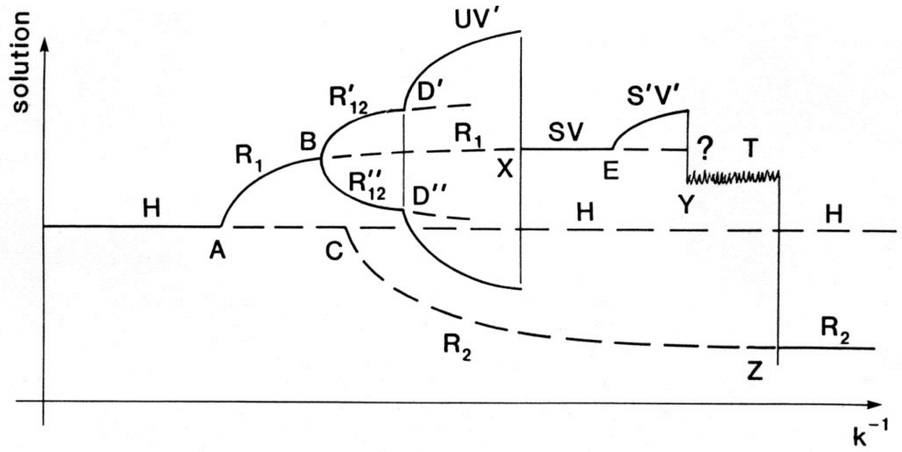
## Bifurcation diagram

**General situation**

$u_t = N(u;\mu)$

$N(u_o;\mu_o) = 0.$

**1) If $L_o = N/\partial u$ at $(u_o;\mu_o)$ is nonsingular, then a unique branch of solutions $u = u(\mu)$ through it exists and is given by $u \cong u_o + (\partial u/\partial\mu)\big|_{\substack{u=u_o \\ \mu=\mu_o}}$.**

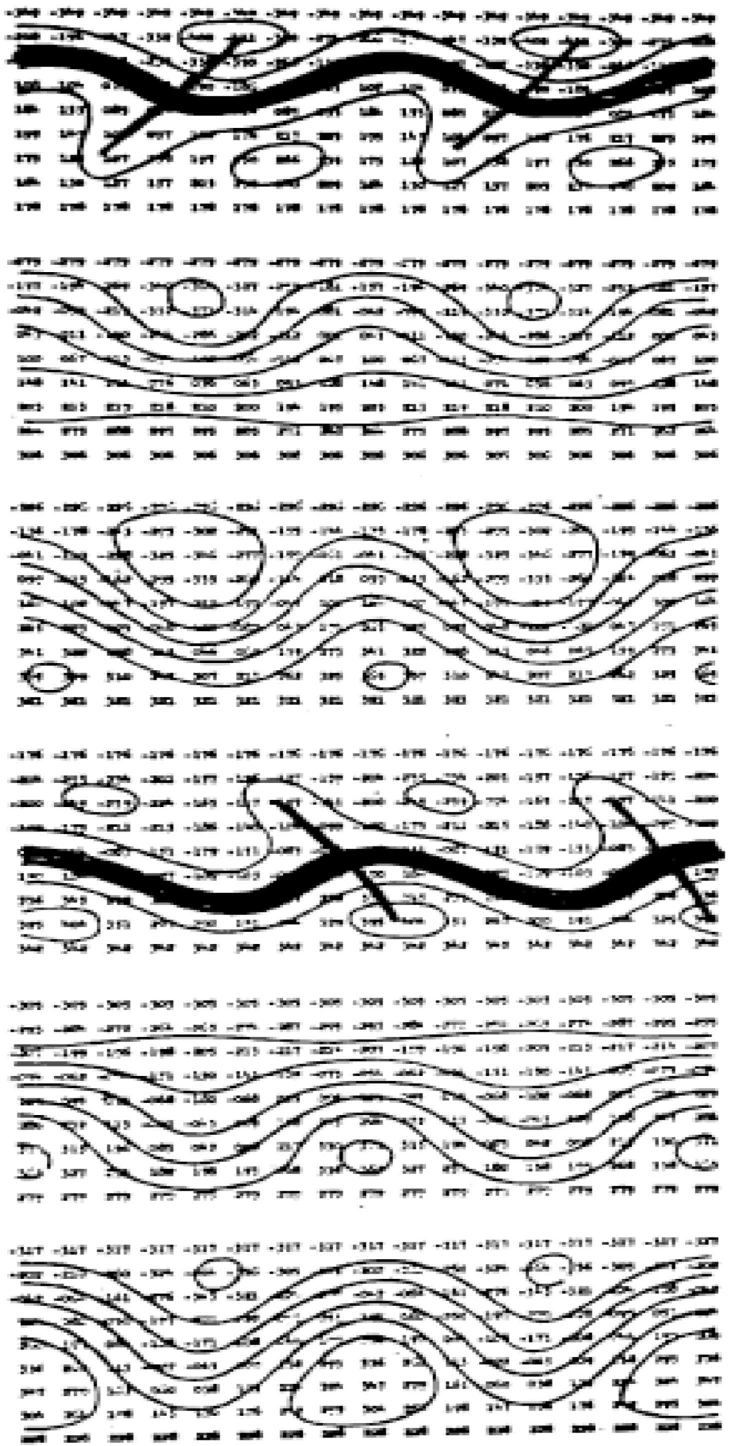



**2) The points at which $det\ L_o = 0$ (i.e., where the Implicit Function Theorem fails) are called bifurcation points, and they are in general isolated. Near such points, the behavior of (2 or more) solutions is parabolic:**

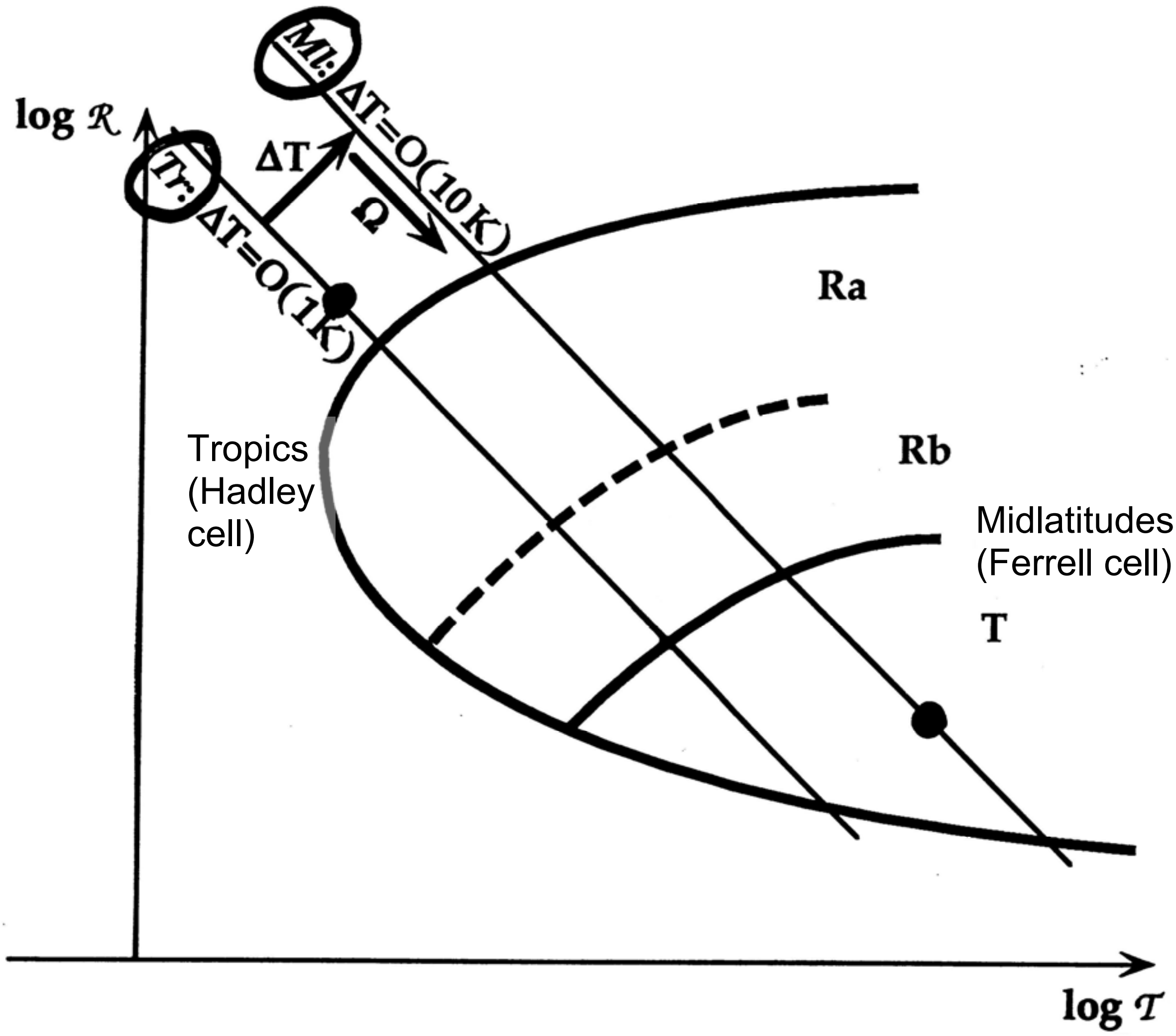
$u - u_o \sim (\mu - \mu_o)^{1/2}$

# Lorenz (1963b) model: Mechanics of vacillation – Index cycle and Shape vacillation

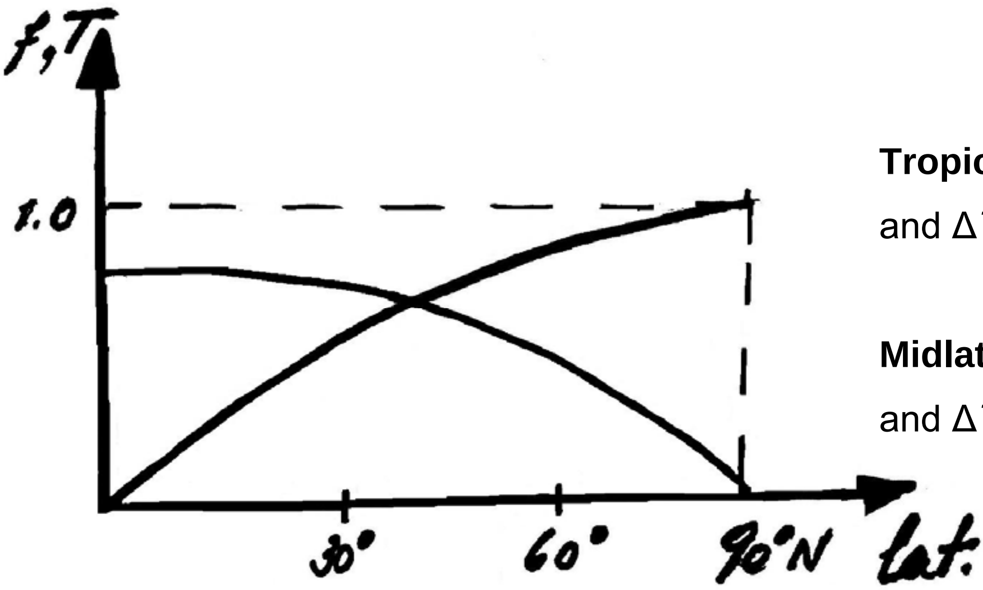
- The model reproduces certain observed aspects of the atmospheric index cycle: latitudinal displacement but not intensity variations.
- It also reproduces certain aspects of tilted-trough (or shape) vacillation in the rotating, differentially heated annulus.



# Rotating annulus & Earth's atmosphere



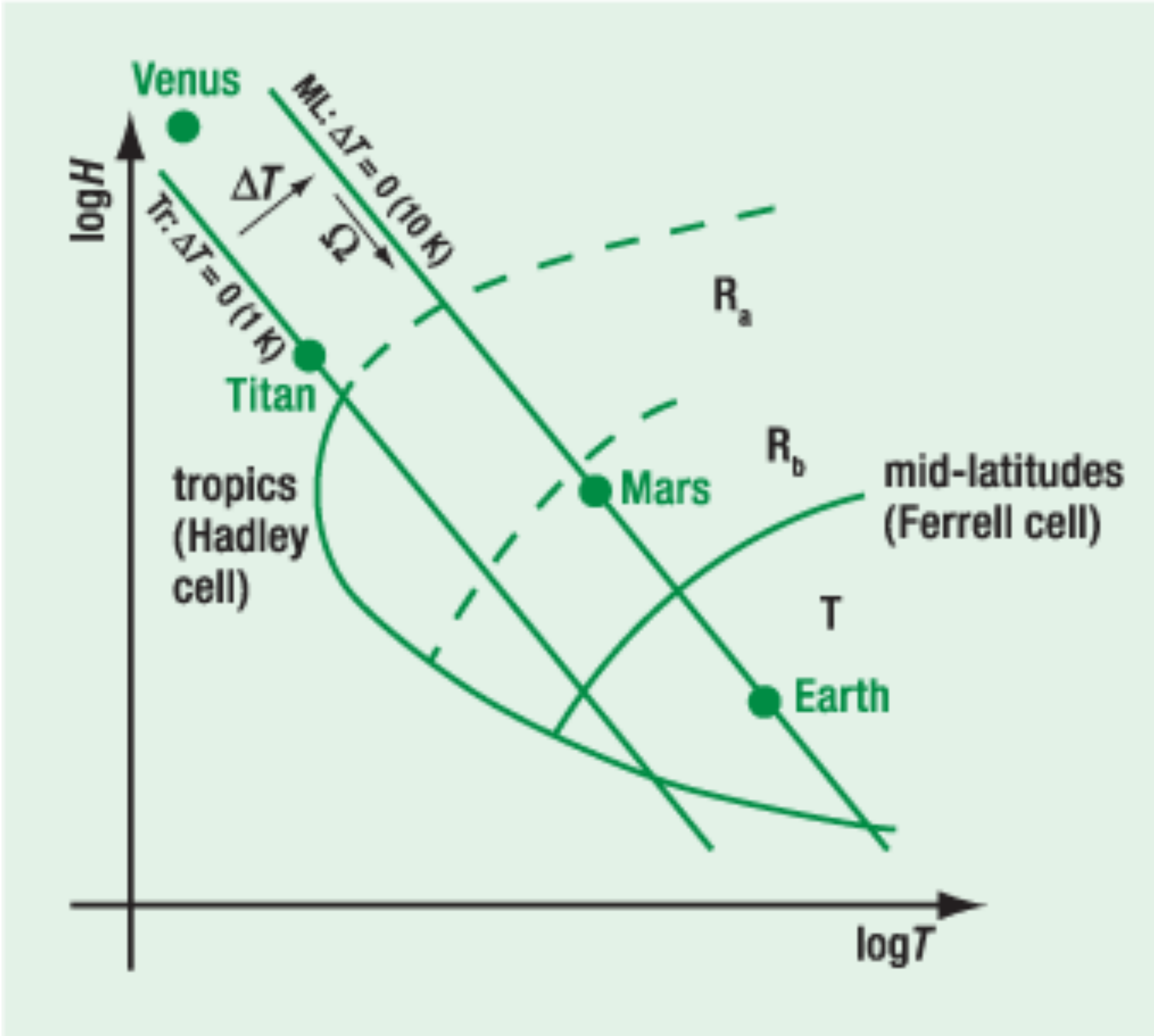

Or why doesn't the Hadley cell on Earth extend to the poles, like on Venus ?




**Tropics :** both $f$ (i.e., Ω) and Δ$T$ small

**Midlatitudes :** both Ω and Δ$T$ large

# Dynamical systems and comparative planetology, II




The tentative place of Earth, Mars, Venus & Titan in this scheme of things

# GFD, bifurcations and chaos

***Problem 3***: Read the paper listed below and report to the class on its contents.

Ghil, M., P. L. Read and L. A. Smith, 2010: Geophysical flows as dynamical systems: The influence of Hide's experiments, *Astron. Geophys*., **51**(4), 4.28–4.35

Calm in the face of chaos …

But just wait till we bring

in randomness, too!

***Calm in the face of chaos …***

***But just wait till we bring***

***in randomness, too!***

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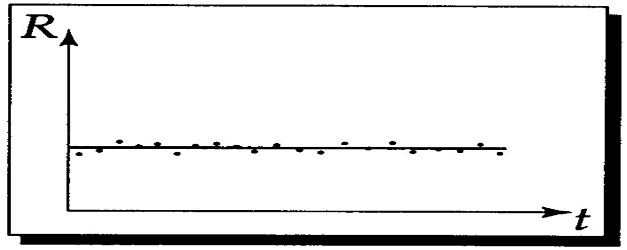
# Prediction and Predictability

**1. Easiest to predict:**

**constant phenomena**

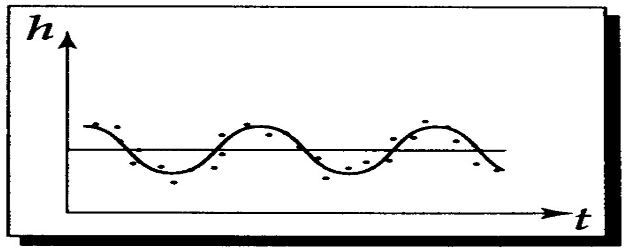
*e.g.*, the radius of the Earth *R* –
only need 1 number




**2. A little harder:**

**periodic phenomena**

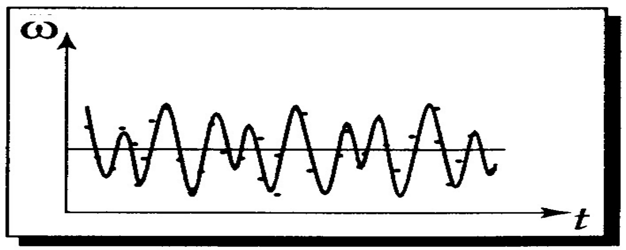
*e.g.*, sunrise, tides – only need 3 numbers :
period, amplitude & phase.




**3. Even harder:**

**multi-periodic phenomena**

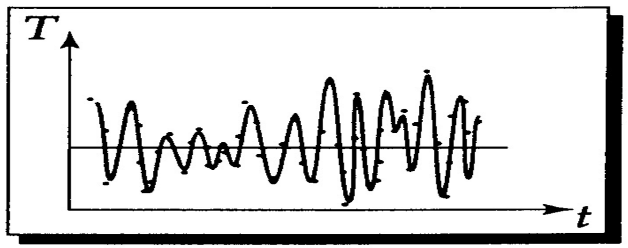
*e.g.*, celestial mecanics –
need (finitely) many numbers




**4. Hardest:**

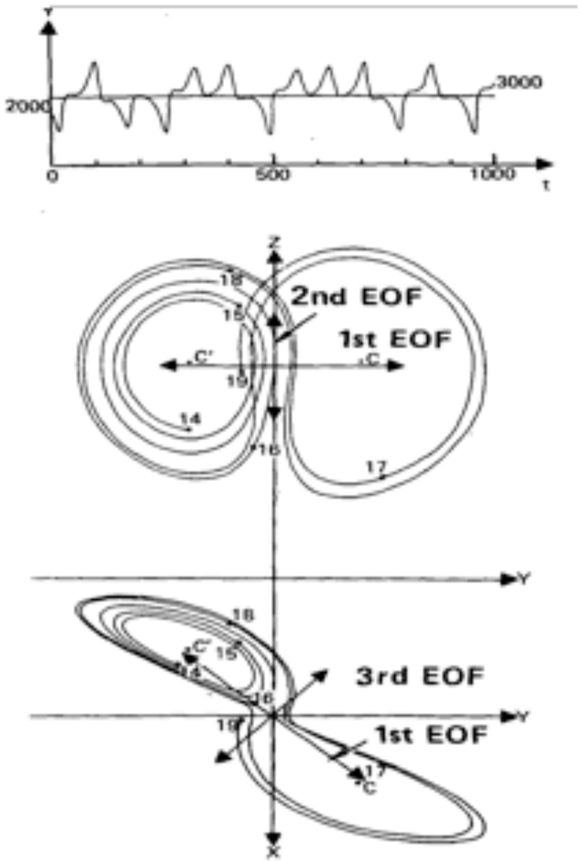
**aperiodic phenomena**

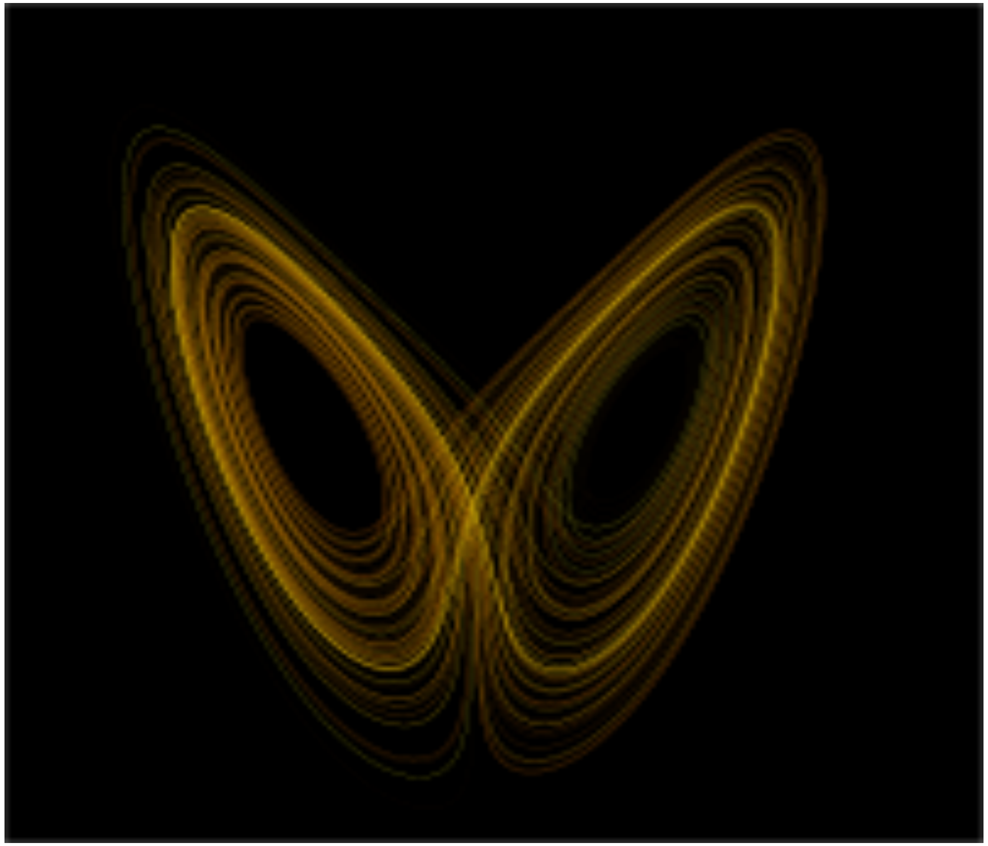
*e.g.*, thermal convection, weather –
infinitely many numbers




➡ The more complex the phenomenon, the harder it is to predict.

# The Lorenz convection (1963a) model
# – some numerical solutions



Plot of $Y = Y(t)$ + projections onto the $(X, Y)$ & $(Y, Z)$ planes

Trajectory in phase space

Both for the canonical "chaotic" values $\rho = 28$, $\sigma = 10$, $\beta = 8/3$.

# The Lorenz (1963a) convection model

***Problem 4***: Find the appropriate software to compute the statistics
of the Lorenz "butterfly" – e.g., pdf, EOFs – and use it to do so.

*Glossary*

pdf = probability density function

EOF = empirical orthogonal function

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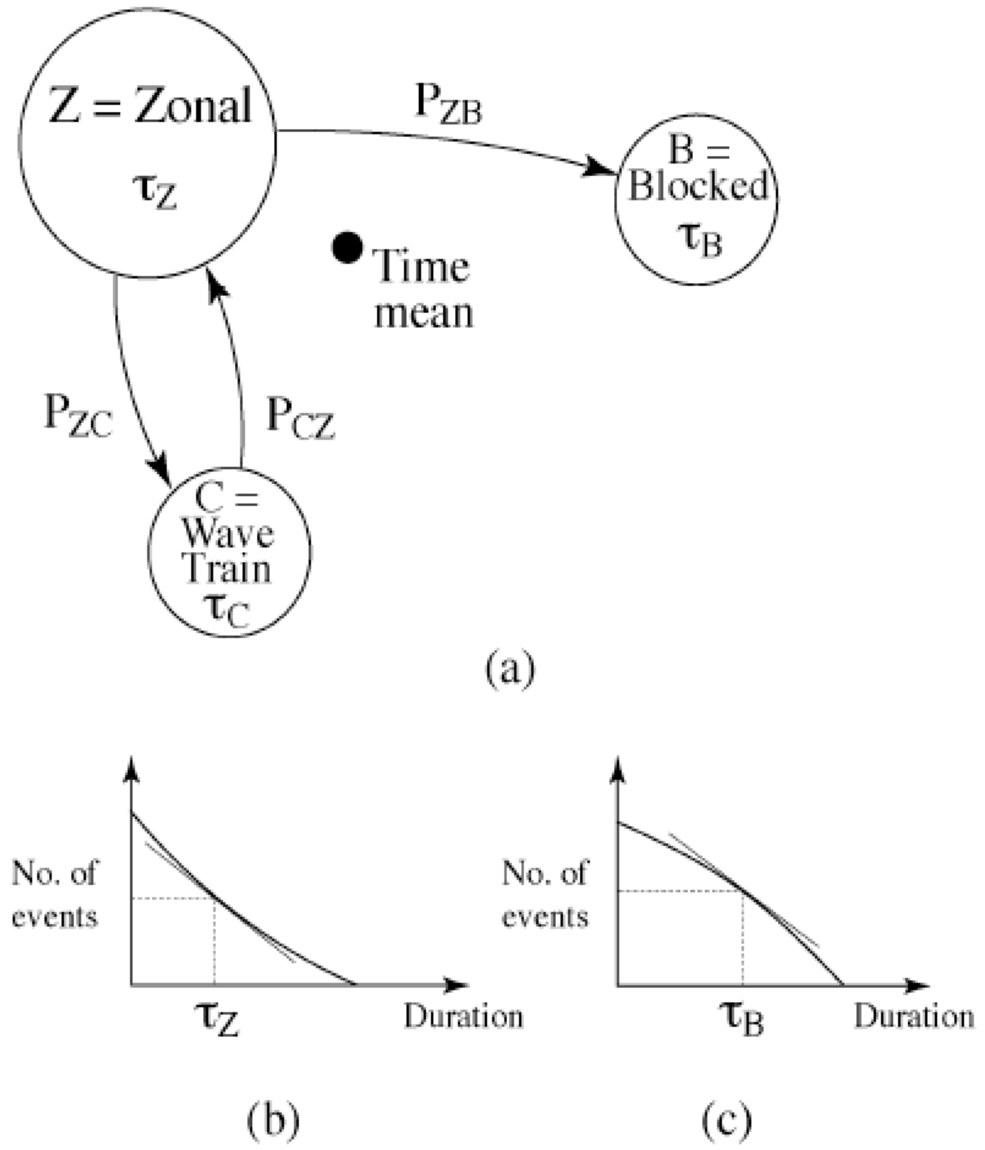
# ***Coarse-graining*** *Markov-chain description of LFV*

1. Reduce the number of degrees of freedom to the most important ones – highest variance.
2. Describe the dynamics in this reduced subspace.



# Multiple Flow Regimes

## A. Classification schemes

1) By position

(i) *Cluster analysis*

– categorical

– NH, Mo & Ghil (1988, *JGR*) – fuzzy

– NH + sectorial, Michelangeli et al. (1995, *JAS*) – hard (*K*–means)

– hierarchical

– NH + sectorial, Cheng & Wallace (1993, *JAS*)

(ii) *PDF estimation*

– univariate

– NH, Benzi *et al*. (1986, *QJRMS*); Hansen & Sutera (1995, *JAS*)

– multivariate

– NH, Molteni *et al*. (1990, *QJRMS*); Kimoto & Ghil (1993a, *JAS*)

– NH + sectorial, Kimoto & Ghil (1993b, *JAS*); Smyth *et al*. (1999, *JAS*)

After Ghil & Robertson (2002, *PNAS*)

# Multiple Flow Regimes **(continued)**

## A. Classification schemes **(continued)**

2) By persistence

(iii) *Pattern correlations*

– NH, Horel (1985, *MWR*)

– SH, Mo & Ghil (1987, *JAS*)

(iv) *Minima of tendencies*

– Models: Legras & Ghil (1985, *JAS*); Mukougawa (1988, JAS); Vautard & Legras (1988, *JAS*)

– Atlantic- European sector : Vautard (1990, *MWR*)

## B. Transition probabilities

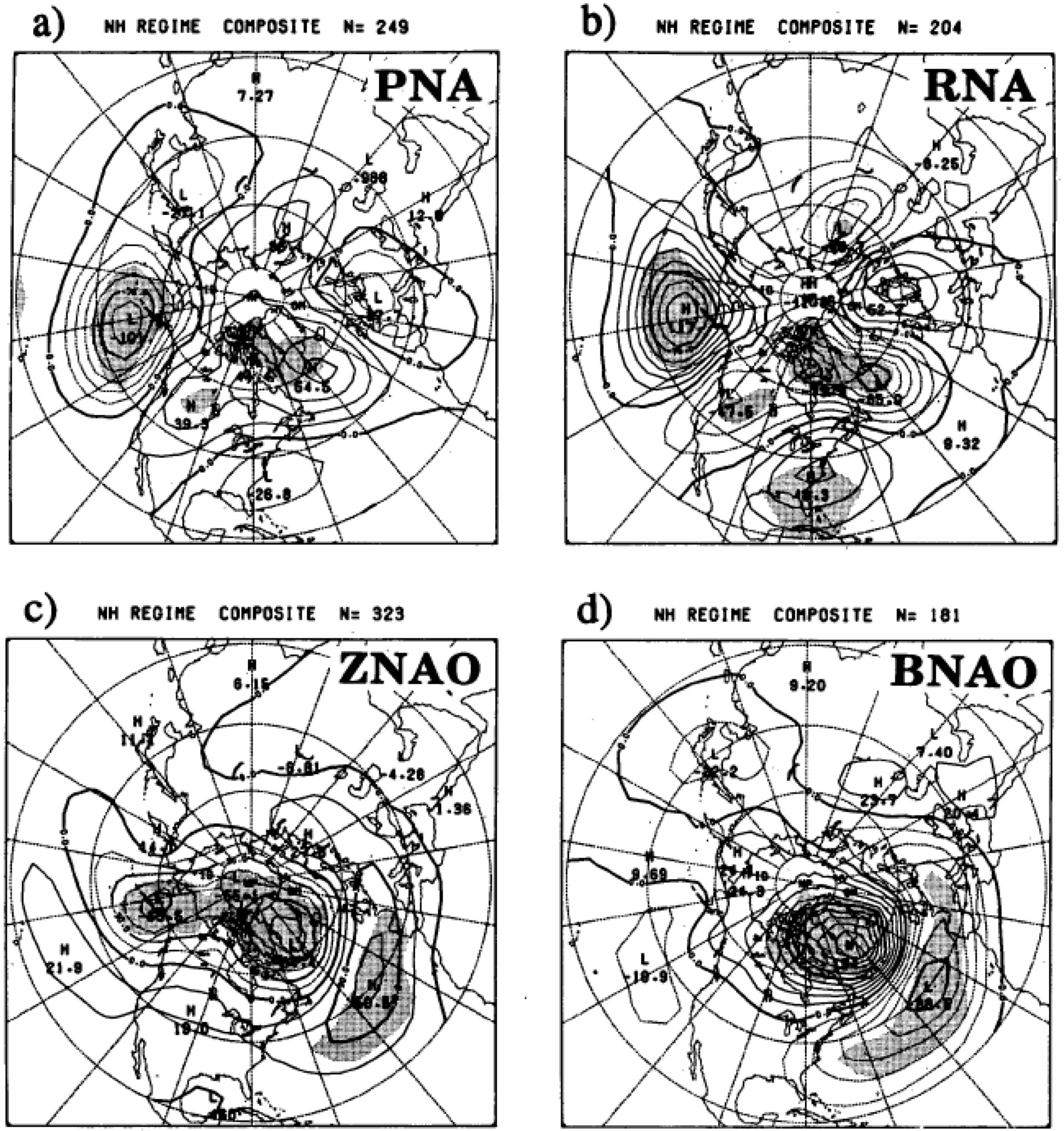
(i) Model & NH – counts (Mo & Ghil, 1988, *JGR*)

(ii) NH & SH – Monte Carlo (Vautard *et al*., 1990, *JAS*)

After Ghil & Robertson (2002, *PNAS*)

# *Multiple Flow Regimes*
# – lowest common denominator, I

Four regimes:
blocked vs. zonal,
in the Pacific–North-American (PNA) & the Atlantic-European sector, respectively
(Kimoto & Ghil, *JAS*, 1993a)



# *Multiple Flow Regimes*
# – lowest common denominator, II

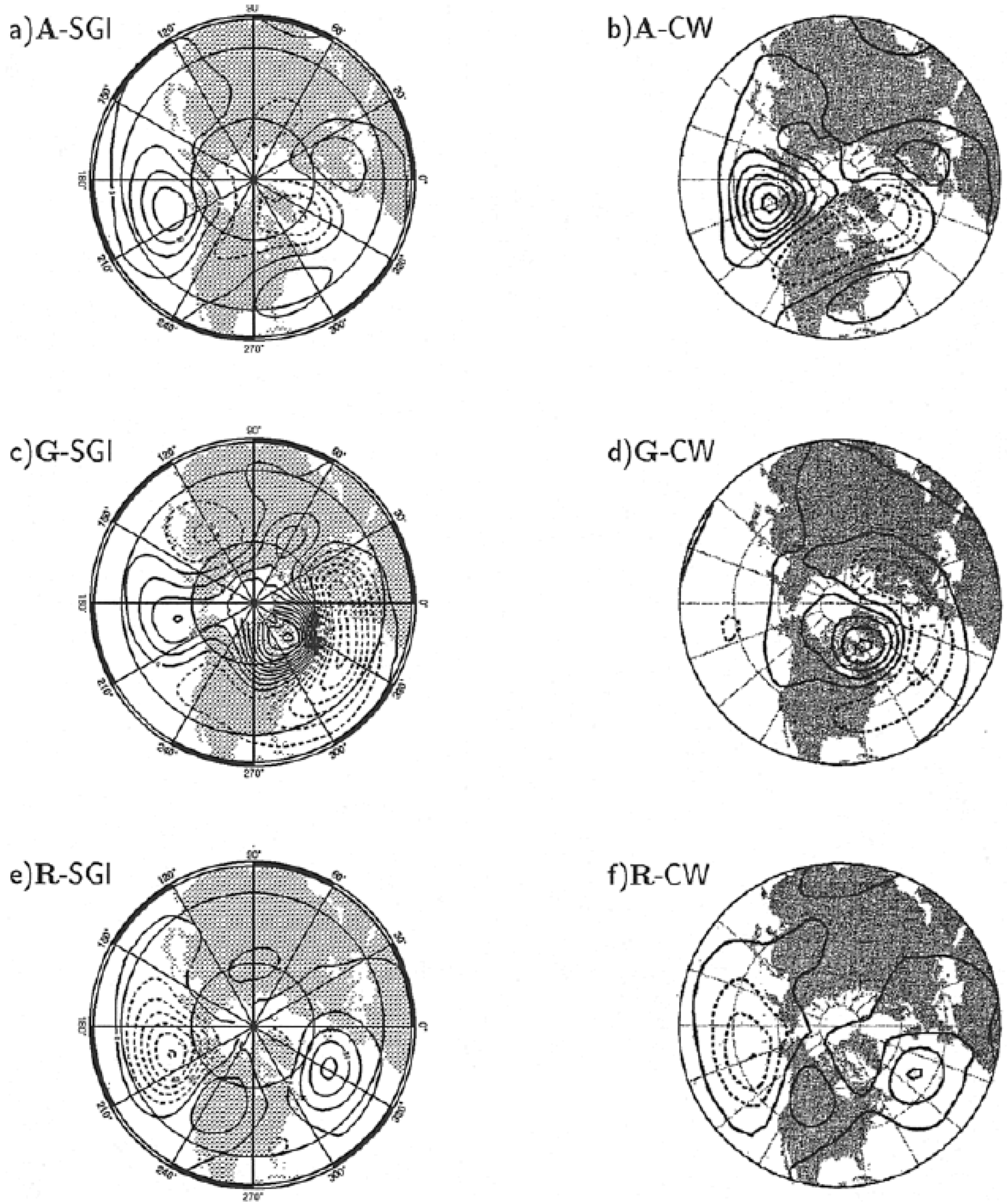
Cheng & Wallace (*JAS*, 1993; **CW**) &. Smyth, Ghil & Ide (*JAS*, 1997; **SGI**) agree well on 3 of the 4 regimes in Kimoto & Ghil (*JAS*, 1993a; **KG**):

**A** – Gulf of Alaska ridge ~ KG's **RNA**

**G** – high over Greenland ~ KG's **PNA**

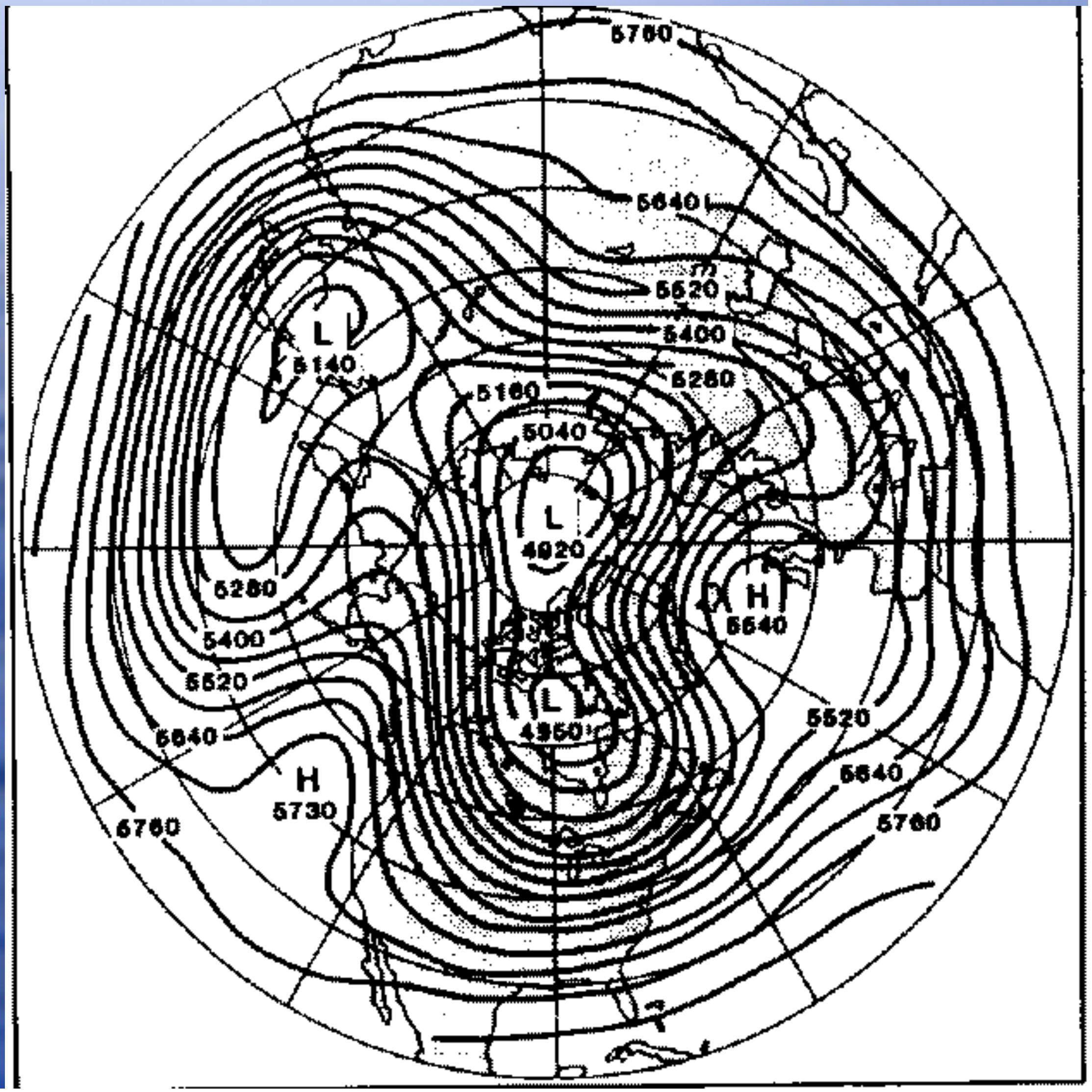
**R** – enhanced ridge over Rockies ~ **BNAO**

SGI's sectorial analyses also find KG's **ZNAO** to be quite robust.



# Blocking: a paradigm of persistent anomaly

Bauer, Namias, Rex and many others noticed the recurrence and persistence of blocking. J. Charney decided to go beyond "talking about it," and actually "do something about it."




Monthly mean 500-hPa map for January 1963 (from Ghil & Childress, 1987)

# **Transitions** Between **Blocked** and **Zonal** Flows

## in a Rotating Annulus with Topography

**Zonal Flow**
13–22 Dec. 1978

**Blocked Flow**
10–19 Jan. 1963

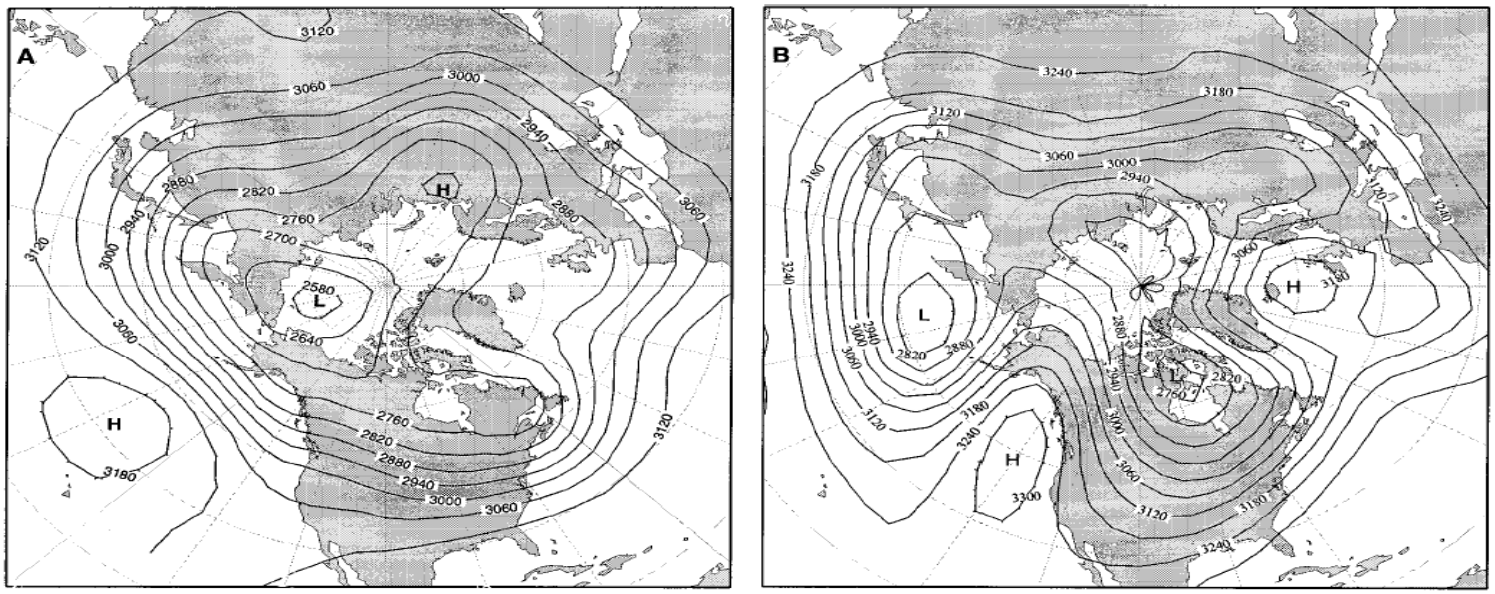


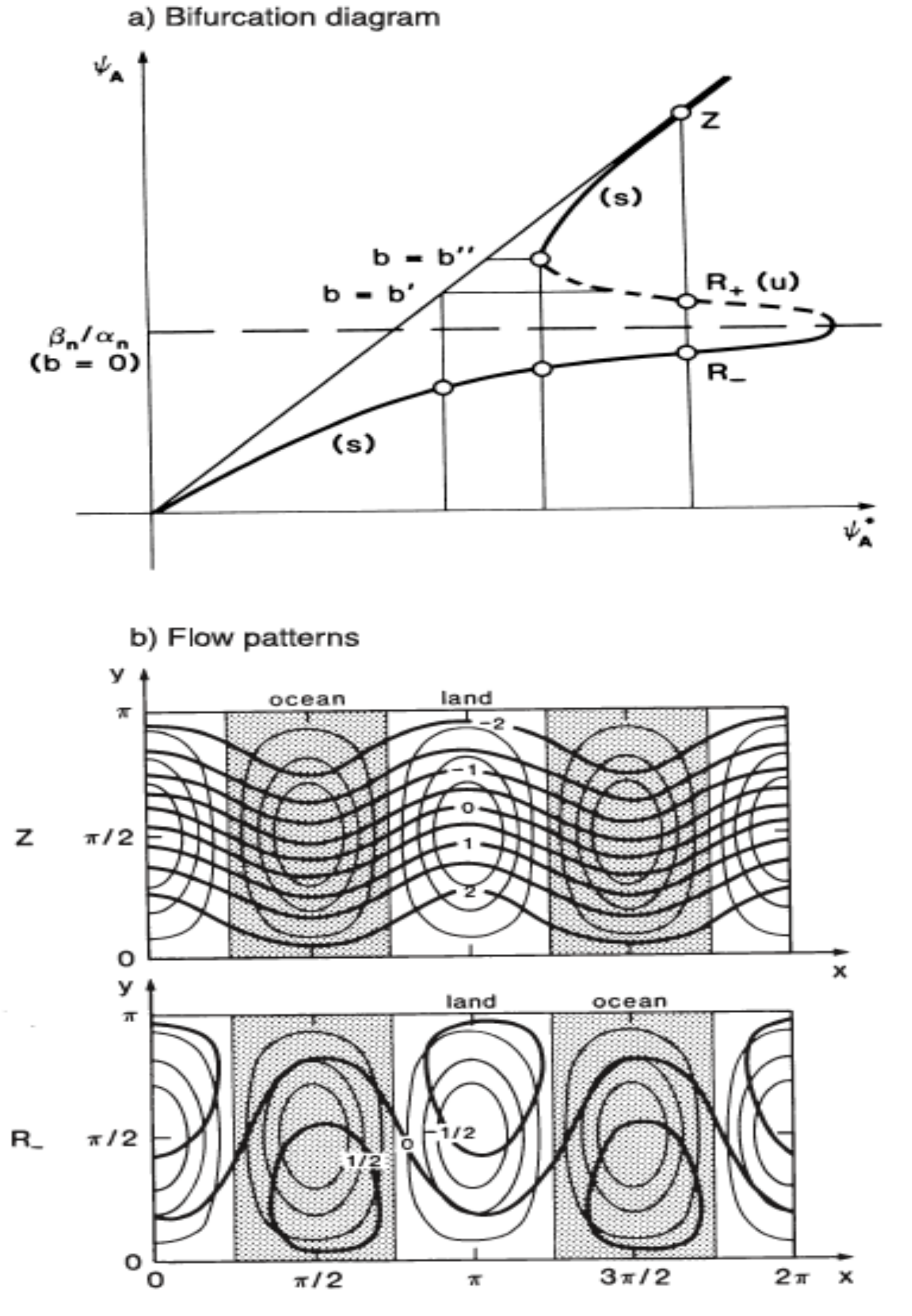

**Fig. 1.** Atmospheric pictures of (**A**) zonal and (**B**) blocked flow, showing contour plots of the height (m) of the 700-hPa (700 mbar) surface, with a contour interval of 60 m for both panels. The plots were obtained by averaging 10 days of twice-daily data for (A) 13 to 22 December 1978 and (B) 10 to 19 January 1963; the data are from the National Oceanic and Atmospheric Administration's Climate Analysis Center. The nearly zonal flow of (A) includes quasi-stationary, small-amplitude waves (32). Blocked flow advects cold Arctic air southward over eastern North America or Europe, while decreasing precipitation in the continent's western part (26).

E.R. Weeks, Y. Tian, J. S. Urbach, K. Ide, H. L. Swinney, & M. Ghil, 1997: *Science*, **278**, 1598–1601.

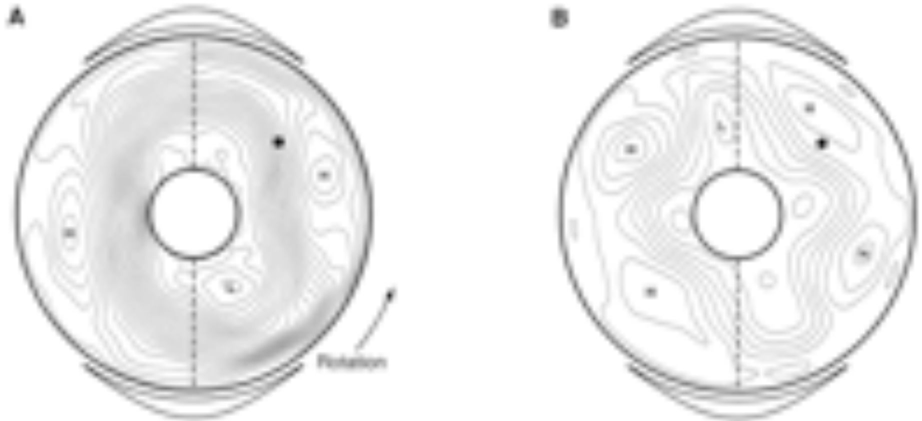
# A toy model for blocking *vs.* zonal flow

- Quasi-geostrophic flow in a mid-latitude β-channel, with 3-mode truncation (zonal + 1 wave).
- Topographic resonance leads to multiple equilibria: zonal + blocked.
- Much criticized as "unrealistic."

Charney & DeVore, 1979: *J. Atmos. Sci.*, **36**, 1205–1216.

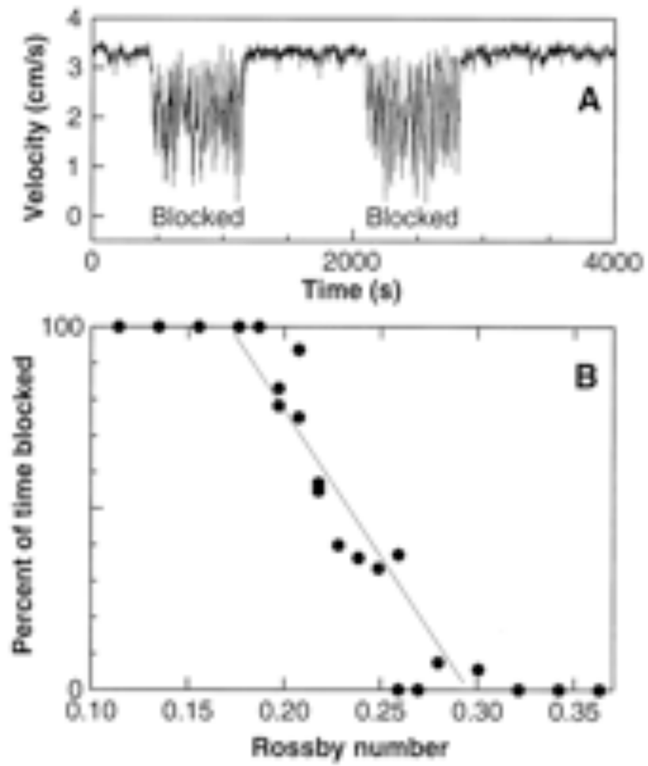


# A little detour via the "barotropic" annulus




Zonal vs. Blocked

streamfunction contours




Relative duration of blocked events

Compare Legras & Ghil (*JAS*, 1985)

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# *Waves vs. Particles: A Pathway to Prediction?*

Is predicting as hard

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No, it's actually quite easy:

Just flip a coin or roll a die!

What's difficult, though, is

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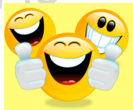
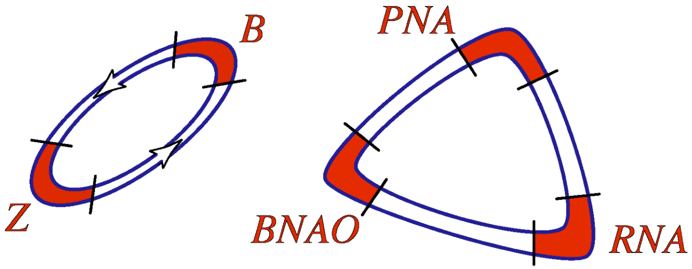


*That's where a little understanding of what we're trying to predict helps!*

Based on Ghil & Robertson (2002)

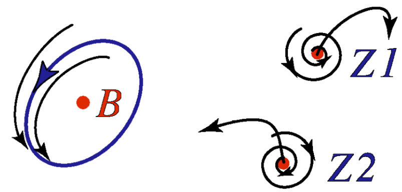
"Waves vs. Particles"
in Atmospheric Low-Frequency Variability

1. Are the regimes but slow phases of the oscillations?




Kimoto & Ghil (*JAS*, 1993a, b)

2. Are the oscillations but instabilities of particular equilibria?

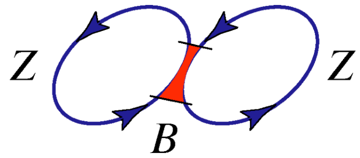



Legras & Ghil (*JAS*, 1985)

3. How about both: "chaotic itinerancy" (Itoh & Kimoto, *JAS,* 1999)

4. How about neither? Null hypotheses:

a) It's all due to interference of linear waves, *e.g.*, neutrally stable Rossby waves;




Lindzen *et al.* (*JAS*, 1982)

b) It's all due to red noise — Hasselmann (*Tellus*, 1976), Mitchell (*Quatern. Res.*, 1976), Penland & co. (Magorian, Sardeshmukh, 1990s).

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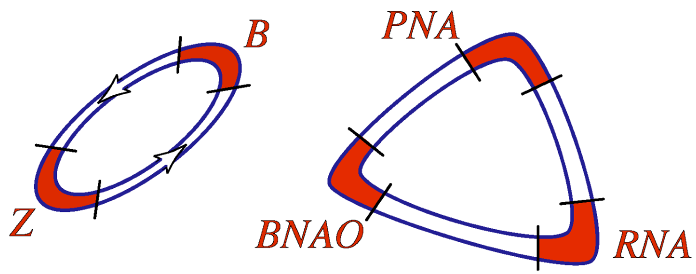


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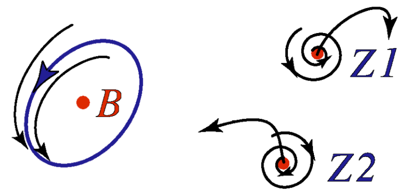


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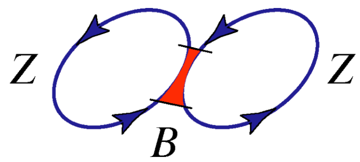


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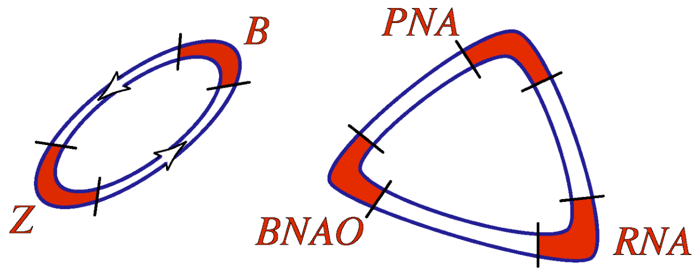


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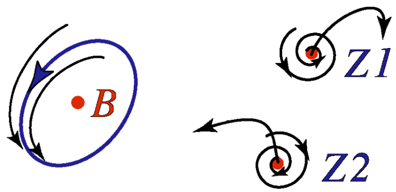


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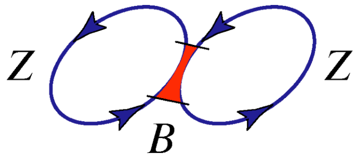


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# Some general references


Eckmann, J.-P., & D. Ruelle, 1985: Ergodic theory of chaos and strange attractors, *Rev. Mod. Phys.*, **57**, pp. 617–656 and 1115.

Ghil, M., R. Benzi, & G. Parisi (Eds.), 1985: *Turbulence and Predictability in Geophysical Fluid Dynamics and Climate Dynamics*, North-Holland, 449 pp.

---------, & S. Childress, 1987: *Topics in Geophysical Fluid Dynamics: Atmospheric Dynamics, Dynamo Theory and Climate Dynamics*, Springer-Verlag, New York, 485 pp.

---------, P. L. Read, & L. A. Smith, 2010: Geophysical flows as dynamical systems: the influence of Hide's experiments. *Astron. Geophys*., **51**(4), 4.28–4.35.

---------, A. Groth, D. Kondrashov, & A. W. Robertson, 2018: Extratropical sub-seasonal–to–seasonal oscillations and multiple regimes: The dynamical systems view, in *The Gap Between Weather and Climate Forecasting: Sub-Seasonal to Seasonal Prediction*, A.~W. Robertson & F. Vitart (eds.), Elsevier, pp. 119–142.

Guckenheimer, J., & P. Holmes, 2002: *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, 2nd ed., Springer-Verlag, New York/Berlin.

Hide, R., 1953: Some experiments on thermal convection in a rotating liquid, *Quart. J. Roy. Meteorol. Soc*., **79**, 161.

-----------, & P. J. Mason, 1975: Sloping convection in a rotating fluid. *Adv. Phys*., 24 , 45–100.

Lorenz, E. N., 1963a: Deterministic nonperiodic flow. *J. Atmos. Sci.*, **20**, 130–141.

------------------,1963b: The mechanics of vacillation. *J. Atmos. Sci.*, **20**, 448–464.

------------------, 1967: *The Nature and Theory of the General Circulation of the Atmosphere*, World Meteorological Organization, Geneva, Switzerland, 161 pp.

Ruelle, D., and F. Takens, 1971: On the nature of turbulence. *Commun. Math. Phys.*, **20**, 167–192.

Weeks, E. R., Y. Tian, J. S. Urbach, K. Ide, H. L. Swinney, and M. Ghil, 1997: Transitions between blocked and zonal flows in a rotating annulus with topography. *Science*, **278**, 1598–1601.

# Reserve slides

## *Lecture I: Outline*

1. General introduction and bibliography
   - Scale dependence of atmospheric & oceanic flows
   - Turbulence & predictability
2. Basic facts of large-scale atmospheric life
   - The atmospheric heat engine
   - Shallowness
   - Rotation
3. Flow regimes, bifurcations & symmetry breaking
   - The rotating, differentially heated annulus
   - Regime diagram & transitions

## *Lecture II: Outline*

1. Observations of persistent anomalies
   – Blocked & zonal flows
   – Characteristics of persistent anomalies
2. The deterministic chaos paradigm
   – Forced dissipative systems
   – Succesive bifurcations
   – Predictability and prediction
3. "Waves" vs. "particles"
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